Chapter 2

Phenomena in Semiconductors
Recombination processes

Equilibrium state:

- $g_T$ – rate of electron-hole pairs thermal generation
- $r_T$ – rate of electron-hole pairs thermal annihilation

$g_T = r_T$

Steady state
constant carrier concentrations

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Non-equilibrium state:

\[ g_T - \text{rate of electron-hole pairs thermal generation} \]
\[ g_r - \text{rate of electron-hole pairs radiative generation} \]
\[ r - \text{rate of electron-hole pairs annihilation} \]

Steady state:
constant carrier concentrations

\[ g_r + g_T = r \]
Recombination processes

Non-equilibrium state:

\[ g_T - \text{rate of electron-hole pairs thermal generation} \]

\[ r - \text{rate of electron-hole pairs anihilation} \]

Transient state

\[ g_T < r \]

\[ R = r - g_T \]

\( R \) – recombination rate

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Recombination processes

\[ R = - \frac{dn}{dt} = \frac{\Delta n}{\tau} \]

\[ \Delta n = \Delta n_0 \exp (-t/\tau) \]

\[ \Delta n(3\tau) = 0.05\Delta n_0 \]

\[ n = n_0 + \Delta n \]

\[ \tau \text{- lifetime} \]
Recombination processes

- **Phonon Recombination** – the whole energy $W_g$ is taken by the thermal vibration of atoms in the lattice, called phonons ($R_{ph}, \tau_{ph}$)

- **Auger (impact) Recombination** – the whole energy $W_g$ is taken by third carrier, electron or hole, called the hot carrier ($R_A, \tau_A$)

- **Radiative Recombination** – the whole energy $W_g$ is taken by a new created photon that can leave the crystal e.g. as the visible radiation ($R_r, \tau_r$)

$$R = R_{ph} + R_A + R_r$$
Phonon recombination – SRH model

Model constants:
- $\tau_{p0}, \tau_{n0}$ - efficiency of $B_2$ and $A_1$
- $n_1, p_1$ - position of $W_t$ inside $W_g$

$W_t$ – recombination centre level

$$R = \frac{(np - n_i^2)}{\tau_{p0}(n + n_1) + \tau_{n0}(p + p_1)}$$

$$\tau_f = \tau_{p0} \frac{n_0 + n_1 + \Delta n}{n_0 + p_0 + \Delta n} + \tau_{n0} \frac{p_0 + p_1 + \Delta n}{n_0 + p_0 + \Delta n}$$

$$n_1 = N_c \exp\left(\frac{W_c - W_t}{kT}\right)$$
$$p_1 = N_v \exp\left(\frac{W_t - W_v}{kT}\right)$$
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Phonon recombination – SRH model

\[ \tau_f = \tau_{p0} \frac{n_0 + n_1 + \Delta n}{n_0 + p_0 + \Delta n} + \tau_{n0} \frac{p_0 + p_1 + \Delta n}{n_0 + p_0 + \Delta n} \]

- **low injection level** - \( \Delta n \ll \max(n_0, p_0) \)

**n-type**

\[ \tau_f = \tau_{p0} \frac{n_0 + n_1}{n_0 + p_0} \]

usually \( n_1 = p_1 = n_i \)

**p-type**

\[ \tau_f = \tau_{n0} \frac{p_0}{n_0 + p_0} \]

\( p_0 \gg \max(n_0, n_i) \)

\( n_0 \gg \max(p_0, n_i) \)
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Phonon recombination – SRH model

\[
\tau_f = \tau_{p0} \frac{n_0 + n_1 + \Delta n}{n_0 + p_0 + \Delta n} + \tau_{n0} \frac{p_0 + p_1 + \Delta n}{n_0 + p_0 + \Delta n}
\]

- **high injection level** - \( \Delta n \gg \max(n_0, p_0, n_i) \)

\[\tau_f = \tau_{p0} + \tau_{n0}\]

![Graph showing \( \tau \) vs \( \Delta n/n_0 \) for n type semiconductors](image)
Auger (impact) recombination

**electron-electron-hole process**

\[ r_{eeh} = C_{An} n^2 p \]

- **C\text{An}** – Auger recombination constant for **e-e-h** process

**electron-hole-hole process**

\[ r_{ehh} = C_{Ap} n p^2 \]

- **C\text{Ap}** – Auger recombination constant for **e-h-h** process
Auger (impact) recombination

**Electron-electron-hole process in steady state:**

\[ R_{An} = r_{eeh0} - g_{eeh0} = 0 \]

\[ g_{eeh0} = r_{eeh0} = C_{An}n_0^2p_0 \]

**Electron-hole-hole process in steady state:**

\[ R_{Ap} = r_{ehh0} - g_{ehh0} = 0 \]

\[ g_{ehh0} = r_{ehh0} = C_{Ap}n_0p_0^2 \]
Auger (impact) recombination

Recombination rate in the electron-electron-hole process:

\[ R_{An} = r_{eeh} - g_{eeh0} = C_{An}n^2p - C_{An}n_0^2p_0 \]

Recombination rate in the electron-hole-hole process:

\[ R_{Ap} = r_{ehh} - g_{ehh0} = C_{Ap}np^2 - C_{Ap}n_0p_0^2 \]
Auger (impact) recombination

- General expression:

\[ R_A = C_{An} \left[n^2 p - n_0^2 p_0\right] + C_{Ap} \left[np^2 - n_0 p_0^2\right] \]

- **Low injection level**

\[ \tau_A = \tau_{An} = \frac{1}{C_{An} n_0^2} = \frac{1}{C_{An} N_D^2} \]

\[ \tau_A = \tau_{Ap} = \frac{1}{C_{Ap} p_0^2} = \frac{1}{C_{Ap} N_A^2} \]
Auger (impact) recombination

- General expression:

\[ R_A = C_{An} \left[ n^2 p - n_0^2 p_0 \right] + C_{Ap} \left[ np^2 - n_0 p_0^2 \right] \]

- **high injection level**

\[ \Delta n >> \max(n_0, p_0) \]

\[ \tau_A = \tau_{As} = \frac{1}{(C_{An} + C_{Ap})(\Delta n)^2} = \frac{1}{C_A (\Delta n)^2} = \frac{1}{C_A n^2} \]
Radiative recombination

\( h\nu \) – photon – quant of radiative energy
\( \nu \) – frequency of emitted electromagnetic waves determining the colour of emitted light

Radiative recombination gives almost monochromatic radiation with the colour depended on \( W_g \)
Surface recombination

At the surface, the number of recombination centres responsible for phonon recombination is larger than in the whole volume due to the larger number of defects and outside agents interference.

As result, in the border layer of semiconductor structure, the recombination rate, $R(x)$ increases ($R_2=\text{var.}$) in comparison to its value inside the structure where usually it is constant ($R_1=\text{const.}$),
Surface recombination

When the excess carriers, $\Delta n$ and $\Delta p$, occur in the semiconductor structure, their recombination is faster in the border layer than in the whole volume disrupting their homogeneous distribution.

As result, the diffusion current of electron-hole pairs from inside towards the border layer appears.

Since the electron-hole pair is electrically neutral, its move gives no electrical current, lowering the inside excess carrier concentration only.
Surface recombination

The border layer essentially influence the recombination process in the whole structure accelerating it since part of electron-hole pairs move to it and recombines there.

If the recombination is to be described properly, the stream of electron-hole pairs toward border must be taken into account.

It can be done by considering $R_2 = \text{var}$ or by introduction of surface recombination model.
Surface recombination

The surface recombination concept consists in the assumption that in the border layer no changes in the recombination rate take place and the electron-hole pair stream towards the border is caused by annihilation of excess electron-hole pairs at the border.

The surface recombination rate, $R_s$, presents the density of excess electron-hole pairs stream flowing towards the surface and captured by it.

It is treated as a new parameter describing the features of semiconductor surface.
Surface recombination

The surface recombination rate is defined by the expression:

\[ R_s = s \Delta n_s \]

\( \Delta n_s \) – excess carrier concentration at the surface [cm\(^{-3}\)]

\( s \) – coefficient of surface recombination [cm/s]

Coefficient \( s \) can change in wide range depending on the surface state, e.g. in Ge:

from \( 10^2 \) cm/s – for etched surface
to \( 10^4 \) cm/s – for a surface after sanding
Electron and hole movement in crystal

- Thermal movement of electrons:

  - Chaotic, from collision to collision, average velocity = 0

  There are the collisions both with other electrons and with the defects of crystal (e.g. thermal vibration of atoms in the lattice nodes)

  - $v_{th}$ – instantaneous velocity increasing with the temperature (in Si at the room temperature in the order of $10^7$ cm/s)
  - $\tau_r$ – relaxation time – the average time between two collisions (in Si at the room temperature in the order of $10^{-9}$ s)
Electron and hole movement in crystal

- Thermal movement in presence of electric field:

  - Electric field accelerates electrons
    
    \[ F = qE \quad \Rightarrow \quad a = F/m \quad \Rightarrow \quad v_E = at \]

    \[ \vec{v} = \vec{v}_{th} + \vec{v}_E \]

  - Electric field results in a constant drift velocity:
    
    \[ v_u = \mu E \]

  where: \( \mu \) - mobility

\[ v_E \] – velocity compound in the direction of electric field \( E \)

\[ v_u \] – drift velocity – the average velocity in the direction of electric field \( E \)
Electron and hole movement in crystal

- **Electron drift current:**
  \[ v_{ue} = \mu_n E \]
  \[ J_{un} = qn v_{ue} = qn \mu_n E \]
  \( \mu_n \) – electron mobility

- **Hole drift current:**
  \[ v_{uh} = \mu_p E \]
  \[ J_{up} = qp v_{uh} = qp \mu_p E \]
  \( \mu_p \) – hole mobility
Electron and hole movement in crystal

Ohm law for semiconductors:

\[ J_u = J_{un} + J_{up} = \]
\[ = q(n\mu_n + p\mu_p)E = \]
\[ = \sigma E = E/\rho \]

where:  \( \sigma \) – electrical conductivity
\( \rho \) – electrical resistivity
Electron and hole movement in crystal

- **Electron diffusion current**: Electron stream $S_n$ is proportional to concentration slope:

  $$S_n = -D_n \frac{dn}{dx}$$

  Electron stream $S_n$ creates the electrical current:

  $$J_{dn} = -qS_n = qD_n \frac{dn}{dx}$$
Electron and hole movement in crystal

- **Hole diffusion current:**

Hole stream $S_p$ is proportional to concentration slope:

$$S_p = -D_p \frac{dp}{dx}$$

Hole stream $S_p$ creates the electrical current:

$$J_{dp} = qS_p = -qD_p \frac{dp}{dx}$$

$$J_{dp} = -qD_p \text{grad } p$$
Electron and hole movement in crystal

- **Transport equations:**

  \[ J_{dn} = q \left( n\mu_n E + qD_n \frac{dn}{dx} \right) \]

  \[ J_{dp} = q \left( p\mu_p E - qD_p \frac{dp}{dx} \right) \]

  \[ J_n = q(n\mu_n E + D_n \text{grad } n) \]

  \[ J_p = q(p\mu_p E - D_p \text{grad } p) \]
Electron and hole movement in crystal

- **Continuity equations:**

Let consider changes of carrier densities, \( n \) and \( p \), during \( \Delta t \) inside the area \( \Delta x \) in the presence of recombination \( R \), generation \( g \) and the carriers flow.

Carrier density balance for \( \Delta t \) time interval:

\[
\Delta n = (g - R)\Delta t + \frac{(J_{n2} - J_{n1}) \Delta t}{q \Delta x}
\]

\[
\Delta p = (g - R)\Delta t + \frac{(J_{p1} - J_{p2}) \Delta t}{q \Delta x}
\]
Electron and hole movement in crystal

- **Continuity equations:**

Let consider changes of carrier densities, \( n \) and \( p \), during \( \Delta t \) inside the area \( \Delta x \) in the presence of recombination \( R \), generation \( g \) and the carriers flow.

After both side division by \( \Delta t \):

\[
\frac{\Delta n}{\Delta t} = (g - R) + \frac{1}{q} \left( J_{n2} - J_{n1} \right)
\]

\[
\frac{\Delta p}{\Delta t} = (g - R) - \frac{1}{q} \left( J_{p2} - J_{p1} \right)
\]
Electron and hole movement in crystal

- **Continuity equations:**

\[
\frac{\delta n}{\delta t} = g - R + \frac{1}{q} \text{div} \ J_n
\]

\[
\frac{\delta p}{\delta t} = g - R - \frac{1}{q} \text{div} \ J_p
\]

After the demand \(\Delta x \to \infty\) and \(\Delta t \to \infty\):

\[
\frac{dn}{dt} = (g - R) + \frac{1}{q} \frac{dJ}{dx}
\]

\[
\frac{dp}{dt} = (g - R) - \frac{1}{q} \frac{dJ_p}{dx}
\]
Basic set of semiconductor structure equations

- **Transport equations:**
  
  \[ J_n = q(n \mu_n E + D_n \text{grad } n) \quad J_p = q(p \mu_p E - D_p \text{grad } p) \]

- **Continuity equations:**
  
  \[ \frac{\delta n}{\delta t} = \frac{g}{q} \div J_n - \frac{1}{q} \div J_n \]
  
  \[ \frac{\delta p}{\delta t} = \frac{g}{q} \div J_p - \frac{1}{q} \div J_p \]

- **Poisson equation:**
  
  \[ \text{div } E = \frac{4 \pi}{\varepsilon} q(p - n + N_d - N_a) \]

- **Kirchhoff equation:**
  
  \[ J = J_n + J_p \]
Carrier injection

Let consider the excess carrier transfer, $\Delta n$, through the layer of width $w$ under the conditions shown in the figure.

For these conditions the basic set of semiconductor structure equations can be reduced to the equation:

$$L^2 \frac{d^2 (\Delta n)}{dx^2} = \Delta n$$

where: $L = (D\tau)^{0.5}$ - diffusion length
Carrier injection

For the problem, one can find an analytical solution that gives excess carrier distributions shown below.

$\beta = \frac{j(w)}{j(0)}$ – transport coefficient

- $\beta \approx 1$: $j(0) \approx j(w)$, $L > w$
- $0 < \beta < 1$: $j(0) > j(w)$, $L \approx w$
- $\beta = 0$: $j(w) = 0$, $L < w$
Carrier injection

- The situation, when the excess carriers flow into the layer due to the border concentration of the excess carriers, is called injection of excess carriers. Such a situation we have in the considered layer.
- The excess carriers injected into a layer diffuse across the layer and their density decreases due to the recombination processes.
- The layer can be transparent or opaque for the diffusion current of the injected excess carriers depending on the layer thickness.
- In each place of the layer, the charge neutrality occurs what means that at each point $\Delta n = \Delta p$.
- The integral over the excess carrier distribution defines some stored charge that quantity corresponds to the actual value of the border density $\Delta n_0$. The slope of the distribution determines the diffusion current and any change of the distribution requires an adequate change of the stored charge. This phenomenon is called diffusion capacity.
The p-n junction comes into being when the type of semiconductor changes from n-type into p-type inside the same crystal lattice:
P-n junction

Sharp change in the carrier density at the border leads to the diffusion currents, $J_{de}$ and $J_{dh}$, through the junction plane.
P-n junction

The carriers leave the dopant ions that, as uncompensated, create the space-charge region, SCR, over the junction plane. As result, in the SCR, the electric field, $E$, that develops the drift currents compensating the diffusion ones occurs:
P-n junction – equilibrium state

At the QN/SCR borders:
- carrier concentration is flat – no diffusion current
- no electric field – no drift current

\[ U_{AK} = 0 \]
\[ I_D = 0 \]
P-n junction – equilibrium state

In **SCR area**:
- On the n-side $N_d \gg n_n \Rightarrow Q_n = q(N_d + p_n - n_n) \approx qN_d$
- On the p-side $N_a \gg p_p \Rightarrow Q_p = q(-N_a - n_p + p_p) \approx -qN_a$

**on n-side:**

$$\frac{dE}{dx} = \frac{4\pi}{\varepsilon} Q_n$$

For $x > a$

$$E = \frac{4\pi q}{\varepsilon} N_d x + C$$

$E$ increases linearly
P-n junction – equilibrium state

In SCR area:
- on the n-side $N_d >> n_n \Rightarrow Q_n = q(N_d + p_n - n_n) \approx qN_d$
- on the p-side $N_a >> p_p \Rightarrow Q_p = q(-N_a - n_p + p_p) \approx -qN_a$

On p-side:

$$\frac{dE}{dx} = \frac{4\pi}{\varepsilon} Q_p$$

for $x < b$

$$E = C - \frac{4\pi q}{\varepsilon} N_a x$$

$E$ decreases linearly
P-n junction – equilibrium state

In SCR area:
- on the n-side $N_d >> n_n \Rightarrow Q_n = q(N_d + p_n - n_n) \approx qN_d$
- on the p-side $N_a >> p_p \Rightarrow Q_p = q(-N_a - n_p + p_p) \approx -qN_a$

Voltage between two points:

$$V_{AB} = \int_{B}^{A} E \, dx$$

$$V_D = \int_{a}^{b} E \, dx$$

$V_D$ - diffusion potential
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P-n junction – equilibrium state

Doping profile

Band model

Majority carrier distribution
**P-n junction – reverse bias**

*At the QN/SCR borders:*

- minority carriers concentrations at the border decrease with the applied voltage running to 0

\[
U_{AK} \rightarrow -\infty \\
 np(b) \rightarrow 0 \\
 pn(a) \rightarrow 0
\]
P-n junction – reverse bias

In the SCR area:
- diffusion current $<<$ drift current
- magnitude of drift currents, $J_{ue}$ and $J_{uh}$, limited by the number of carriers

$J_{ue}(b) = J_{de}(b) = \frac{qD_n dn_p(b)}{dx}$

$J_{uh}(a) = J_{dh}(a) = -\frac{qD_n dn_p(b)}{dx}$
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P-n junction – forward bias

At the QN/SCR borders:

- minority carriers concentrations at the border increase with the applied voltage leading to the minority carrier injection

\[ \Delta n_0 \]

\[ U_{AK} > 0V \]

\[ n_p(b) > n_{p0} \]

\[ p_n(a) > p_{n0} \]

\[ V_D - U_{AK} = \int_{a}^{b} E \, dx \]
P-n junction – forward bias

In the QN layers:
- injected minority carriers create diffusion currents in the area close to the SCR, which disappear as result of recombination.
**P-n junction – ideal diode equation**

**Ideal diode I-V characteristics:**

\[
I_D = I_{s0} \left( \exp \left( \frac{qU}{kT} \right) - 1 \right)
\]

- \( I_{s0} \) – saturation current
P-n junction – injection coefficient

Injection coefficient for electrons:

$$\gamma_{e\rightarrow p} = \frac{J_{ej}}{J}$$

Injection coefficient for holes:

$$\gamma_{h\rightarrow n} = \frac{J_{hj}}{J}$$